ANALYSIS OF LATERALLY LOADED SHAFTS IN ROCK

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ABSTRACT: The behavior of both flexible and rigid shafts socketed into rock and subjected to lateral loads and moments is studied. Parametric solutions for the load-displacement relations are generated using the finite element technique. Based on these solutions, simple, approximate, closed-form equations are developed to describe the response for the full range of loading conditions, material parameters, and socket-rock mass stiffnesses encountered in practice. These results are in close agreement with available solutions for the limiting cases of flexible and rigid shafts. The solutions give horizontal groundline displacements and rotations and can incorporate an overlying soil layer. The problem of assessing the lateral load capacity of rock-socketed foundations is also addressed, and a method of analysis to predict this capacity is suggested. The application of the theory, in the form of back-analysis, to a single case involving the field loading of a pair of rock-socketed shafts is also described.

INTRODUCTION

Shaft foundations in rock are often used to transmit large lateral (horizontal) forces and overturning moments to the ground, where adequate resistance to this form of loading must be provided. As in most design, an adequate margin of safety against collapse must be ensured, and the displacements resulting from this form of loading must be tolerable. In this paper, some of the existing methods for predicting the lateral displacements of deep foundations will be reviewed briefly, and some new and simple closed-form solutions for the response of rock-socketed shafts will be presented. The problem of assessing the margin of safety of a rock-socketed foundation under lateral loading is also addressed, and a method of analysis to predict the lateral capacity is suggested.

RECENT METHODS FOR PREDICTING LATERAL DEFLECTIONS

In recent years, theoretical approaches for predicting the lateral displacements of long slender piles in soil have been developed extensively. Two main approaches have generally been used. In the simplest, known as the subgrade-reaction method, the laterally loaded pile is idealized as an elastic beam loaded transversely and restrained by uniform linear springs acting along the length of the beam. The effect of this idealization is to ignore the continuous nature of the soil medium. Closed-form solutions for this idealization are available for a variety of loading conditions and end restraints on the pile (Hetenyi 1946). This model has been improved by allowing the spring stiffness to vary along the length of the pile (Reese and Matlock 1956; Matlock and Reese 1960), and, subsequently, by replacing the linear springs by nonlinear p–y–curves (Matlock and Ripperger 1958; Matlock 1970; Reese et al. 1975). For these extended forms of the subgrade-reaction

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approach, numerical solution techniques are required and, from a design point of view, the method loses some of its attraction.

A significant development in the analysis of laterally loaded piles was made by modeling the soil as an elastic continuum and the pile as an elastic beam. Numerical solutions were developed, first with the use of the integral equation (or boundary element) method (Poulos 1971a, 1971b, 1972; Banerjee and Davies 1978) and second with the use of the finite element method (Randolph 1977, 1981). Most of these elastic solutions were presented in the form of charts. Approximate but convenient closed-form expressions for the response of flexible piles to lateral loading have also been published by Randolph (1981).

The designer of laterally loaded flexible piles in soil now has solutions for the pile response that are very simple to use. Unfortunately for the designer of laterally loaded, rock-socketed shafts, these solutions do not cover all cases in practice. Hence, new solutions are presented here to cover shafts socketed into rock.

**PROBLEM IDEALIZATION**

The problem shown in Fig. 1 represents the cases where either rock is at the ground surface or the lateral loading on the shaft at the level of the rock surface can be specified completely. The shaft is idealized as a cylindrical elastic inclusion, with an effective Young's modulus ($E_e$), Poisson ratio ($\nu_c$), depth ($D$), and diameter ($B$). For a solid shaft, having an actual bending rigidity equal to $(EI)_c$, the effective Young's modulus is given by

$$E_e = \frac{(EI)_c}{\pi B^4} \frac{64}{\pi B^4}$$

It is assumed that the elastic shaft is embedded in a homogeneous, isotropic elastic rock mass, with properties $E_r$ and $\nu_r$. At the surface of the rock mass, it is subjected to a known lateral (horizontal) force ($H$) and an overturning moment ($M$).

**ANALYSIS OF LATERAL DEFLECTIONS**

Some simple closed-form expressions are presented for both relatively flexible and relatively rigid shafts subjected to lateral loading. These equa-
tions have been derived from the results of finite element studies of the behavior of axisymmetric bodies subjected to nonsymmetric loading. The technique used has been described by Wilson (1965) and is similar to that used for the study of the lateral loading of flexible piles (Randolph 1977, 1981). Eight-noded, isoparametric, quadrilateral elements, with $3 \times 3$ Gaussian integration, were used in the present investigation. Further details are given by Carter and Kulhawy (1988).

An extensive parametric study has been performed for socketed shafts covering a large range of relative stiffnesses and, following Randolph (1977, 1981), it was found that the effects of variations in the Poisson ratio of the rock mass ($\nu_r$), could be represented approximately by considering an equivalent shear modulus of the rock mass ($G^*$), defined as

$$G^* = G_r \left(1 + \frac{3\nu_r}{4}\right) \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \Quad
For these cases, the shaft response depends only on the modulus ratio \( \frac{E_c}{G^*} \) and Poisson ratio of the rock mass \( (\nu_r) \). Dotted curves corresponding to the equality condition in (4) have been plotted on Figs. 2(b), 3(b), and 4(b), from which it can be seen that the finite element predictions are effectively independent of \( D/B \) whenever (4) holds. Such a shaft is flexible, and the following closed-form expressions that were suggested by Randolph (1981) provide accurate approximations for the deformations:

\[
\begin{align*}
 u & = 0.50 \left( \frac{H}{G^*B} \right) \left( \frac{E_c}{G^*} \right)^{-1/7} + 1.08 \left( \frac{M}{G^*B^2} \right) \left( \frac{E_c}{G^*} \right)^{-3/7} \\
 \theta & = 1.08 \left( \frac{H}{G^*B^2} \right) \left( \frac{E_c}{G^*} \right)^{-3/7} + 6.40 \left( \frac{M}{G^*B^3} \right) \left( \frac{E_c}{G^*} \right)^{-5/7}
\end{align*}
\]

The appropriate forms of these equations have been plotted on Figs. 2(a), 3(a), and 4(a). It is clear from these figures that (5) and (6) provide adequate predictions of the behavior of flexible shafts socketed into elastic rock masses. Randolph (1981) verified their accuracy for the following ranges of parameters: \( 10^2 \leq E_c/E_r \leq 10^6 \) and \( D/B \geq 10 \). The present study has verified that the range of applicability can be extended to \( 1 \leq E_c/E_r \leq 10^6 \) and \( D/B \geq 1 \).

There are cases encountered in practice, particularly when short stubby shafts are socketed into weaker rock, where the shafts will behave as rigid structural members. In these cases, the displacements of the shaft will be independent of the modulus ratio \( (E_c/E_r) \) and will depend only on the slenderness ratio \( (D/B) \) and Poisson ratio of the rock mass \( (\nu_r) \).

The dotted curves in Figs. 2(a), 3(a), and 4(a) indicate that a shaft will behave as a rigid member when

\[
\frac{D}{B} \leq 0.05 \left( \frac{E_c}{G^*} \right)^{1/2} \hspace{1cm} (7a)
\]
FIG. 4. Lateral Load-Rotation and Moment-Displacement Relations
The present study shows that the displacements of these rigid shafts can be expressed, to sufficient accuracy, by these simple closed-form expressions

\[ u = 0.4 \left( \frac{H}{G^*B} \right) \left( \frac{2D}{B} \right)^{-1/3} + 0.3 \left( \frac{M}{G^*B^2} \right) \left( \frac{2D}{B} \right)^{-7/8} \]  

\[ \theta = 0.3 \left( \frac{H}{G^*B^2} \right) \left( \frac{2D}{B} \right)^{-7/8} + 0.8 \left( \frac{M}{G^*B^3} \right) \left( \frac{2D}{B} \right)^{-5/3} \]

Appropriate forms of these equations are plotted as solid curves on Figs. 2(b), 3(b), and 4(b), where satisfactory agreement with the finite element solutions can be seen. Because the shaft displaces as a rigid body in the elastic rock mass, the depth beneath the surface to its center of rotation \( z_c \) can be computed as

\[ \frac{z_c}{B} = \frac{0.4 \left( \frac{2D}{B} \right)^{-1/3} + 0.3 \left( \frac{e}{B} \right) \left( \frac{2D}{B} \right)^{-7/8}}{0.3 \left( \frac{2D}{B} \right)^{-7/8} + 0.8 \left( \frac{e}{B} \right) \left( \frac{2D}{B} \right)^{-5/3}} \]

in which \( e = M/H = \) the vertical eccentricity of the applied horizontal force \( H \). When applying (7)–(10), it should be noted that their accuracy has been verified only for the following ranges of parameters: \( 1 \leq D/B \leq 10 \) and \( E_r/E \geq 1 \).

Traditionally, the influence factors for laterally loaded piles and shafts have been presented in numerous charts. The approximate equations presented here are more attractive for design because of their succinctness.

Shafts can be described as having intermediate stiffness whenever the slenderness ratio is bounded approximately as follows:

\[ 0.05 \left( \frac{E_e}{G^*} \right)^{1/2} < \frac{D}{B} < \left( \frac{E_e}{G^*} \right)^{2/7} \]

Figs. 2–4 show that, in these cases, the finite element predictions are almost always larger than the predictions from (5) and (6) for flexible shafts and (8) and (9) for rigid shafts. Typically, the displacements for an intermediate case exceed the maximum of the predictions for corresponding rigid and flexible shafts by no more than about 25%, and often by much less. For simplicity, without sacrificing much accuracy, it is suggested that the displacements in the intermediate case be taken as 1.25 times the maximum of either: (1) The predicted displacement of a rigid shaft with the same slenderness ratio \( (D/B) \) as the actual shaft; or (2) the predicted displacement of a flexible shaft with the same modulus ratio \( (E_e/G^*) \) as the actual shaft. Values calculated this way should, in most cases, be slightly larger than those given by the more rigorous finite element analysis for a shaft of intermediate stiffness.
Consider now a layer of soil overlying rock as shown in Fig. 5(a). In this problem, it is assumed that the complete distribution of soil reaction on the shaft is known and that the socket provides the majority of resistance to the lateral load or moment. The groundline horizontal displacement ($u$) and rotation ($\theta$) can then be determined after structural decomposition of the shaft and its loading, as shown in Figure 5(b). The portion of the shaft within the soil may be analyzed as a determinant beam subjected to known loading. The displacement and rotation of point $A$ relative to point $O$ can be determined by established techniques of structural analysis (e.g., the slope-deflection method). The horizontal shear force ($H_o$) and bending moment ($M_o$) acting in the shaft at the rock surface level can be computed from statics, and the displacement and rotation at this level can be computed by the methods described previously. The overall groundline displacements can then be calculated by superposition of the appropriate parts.

The key to using this method successfully lies in determining the distribution of the soil reaction. As a worst case, the soil could be ignored completely, allowing the portion of the shaft in soil to be treated as a free-standing cantilever. This approach, however, may be overly conservative. For simplicity, it will be assumed that the magnitude of the lateral loading applied is sufficient to cause yielding within the soil and for limiting soil reaction stresses to develop along the leading face of the shaft. Furthermore, it is assumed that this limiting condition is reached at all points down the shaft, from the ground surface to the interface with the underlying rock mass.

These assumptions may represent an oversimplification because some loading conditions may not be large enough to develop this limiting condition. In these cases, the predictions of groundline displacements will overestimate the true displacements. In many cases, however, the decision to socket the shafts into rock will have been made because of the inability of the soil to provide adequate lateral restraint. Therefore, in these circumstances, the assumption of a limiting soil reaction distribution is likely to be sufficient.

The determination of the limiting soil reactions is discussed as follows for the cases of cohesive soil in undrained ($\varphi = 0$) loading and frictional soil ($c = 0$) in drained loading.
Shafts through Cohesive Soils

It is often accepted that the ultimate soil resistance for piles and shafts in cohesive soil during undrained loading increases with depth from about 2s_u at the surface (s_u = undrained shear strength of the soil) to about 8 to 12s_u at a depth of about 3 foundation diameters below the surface. One commonly used, simplified distribution of soil resistance ranges from zero at the ground surface to a depth of 1.5D and has a constant value of 9s_u below this depth (Broms 1964a). This distribution is illustrated in Fig. 6 and assumes that the shaft movements will be sufficient to generate this reaction distribution.

For this case, the lateral displacement (u_{AO}) and rotation (\theta_{AO}) of point A at the shaft butt, relative to point O at the soil-rock interface, can be determined by structural analysis of the shaft, treated as a beam subjected to known loading, and are given by (Carter and Kulhawy 1988)

\[
(EL)_{c}u_{AO} = \frac{1}{3} HD_{s}^{3} + \frac{1}{2} MD_{s}^{2} - \frac{9}{8} s_u(D_{s} - 1.5B)^{3}(D_{s} + 0.5B)B \quad \text{(12)}
\]

\[
(EL)_{c}\theta_{AO} = \frac{1}{2} HD_{s}^{2} + MD_{s} - \frac{3}{2} s_u(D_{s} - 1.5B)^{3}B \quad \text{............... (13)}
\]

in which \( D_s \) = the depth of the soil layer; and \((EL)_c\) = the bending rigidity of the shaft section.

The shear force \( (H_o) \) and bending moment \( (M_o) \) acting at point O can be determined from statics as

\[
H_o = H - 9s_u(D_{s} - 1.5B)B \quad \text{........................ (14)}
\]

\[
M_o = M - 4.5s_u(D_{s} - 1.5B)^{2}B + HD_{s} \quad \text{........... (15)}
\]

The contribution to the groundline displacement from the loading transmitted to the rock mass now can be computed by analyzing a rock-socketed shaft of embedded length \( D_r \), subjected to a horizontal force \( (H_o) \) and moment \( (M_o) \) applied at the level of the rock surface. This procedure has been described previously. These components of displacement should be added to the displacement and rotation calculated using (12) and (13) to determine the overall groundline response.

\[
H_o = H - 9s_u(D_{s} - 1.5B)B
\]

\[
M_o = M + HD_{s} - 4.5s_u(D_{s} - 1.5B)^{2}B
\]

**FIG. 6. Idealized Loading of Socketed Shaft through Cohesive Soil**
Shafts through Cohesionless Soil

For shafts in cohesionless soil, the behavior can be analyzed using the reaction distribution suggested by Broms (1964b), as shown in Fig. 7. The following assumptions have been made in deriving this distribution:

1. The active soil stress acting on the back of the shaft is neglected.
2. The distribution of soil stress along the projected front of the shaft is equal to three times the Rankine maximum passive stress.
3. The shape of the shaft section has no influence on the distribution of ultimate soil stress or the magnitude of the ultimate lateral resistance.
4. The full lateral resistance is mobilized at the movement being considered.

The distribution of soil resistance $p_u$ at depth $z$ is given by

$$p_u = 3K_p\bar{\sigma}_v \tag{16}$$

in which $\bar{\sigma}_v = \text{vertical effective stress at depth } z$; $K_p = (1 + \sin \phi)/(1 - \sin \phi)$; and $\phi$ = effective stress friction angle of the soil.

The simplifying assumption of an ultimate soil resistance equal to three times the Rankine maximum passive stress is based on limited empirical evidence from comparisons between predicted and observed ultimate loads of piles in sand (Broms 1964b). These comparisons suggest that the assumed factor of 3 may, in some cases, be conservative.

For a dry cohesionless soil, with unit weight $\gamma$, the relative displacement and rotation ($u_{AO}$ and $\theta_{AO}$) can be determined from structural analysis as (Carter and Kulhawy 1988)

$$EI_{u_{AO}} = \frac{1}{3} HD_s^3 + \frac{1}{2} MD_s^2 - \frac{1}{10} K_p \gamma D_s^3 B \tag{17}$$

FIG. 7. Idealized Loading of Socketed Shaft through Cohesionless Soil
The shear force and bending moment at the level of the rock surface are given by

\[ H_o = H - 1.5K_p \gamma D_s^2B \]  \hspace{1cm} (19)

\[ M_o = M - 0.5K_p \gamma D_s^2B + HD_s \]  \hspace{1cm} (20)

The displacements at the groundline can be computed by assuming that the loading given in (19) and (20) acts on a rock-socketed shaft and then adding the resulting displacements to those given by (17) and (18).

**LATERAL LOAD CAPACITY**

The lateral capacity of shafts socketed into rock has received very little attention in the literature. Perhaps the reason is that the lateral design is governed largely by displacement considerations, and, therefore, the capacity has been assigned lesser importance. Whatever the reason, reliable evaluation of the lateral capacity is still important, if only to determine the likely margin of safety existing at working load levels. Theoretically, the problem is very difficult to solve, which also may account for the lack of published work in this area.

An approximate theoretical approach for estimating the ultimate lateral capacity is presented here. It is assumed in the following that the shaft section has sufficient moment and shear capacity to resist the applied loading, and ultimate failure of the shaft occurs when the surrounding rock mass is not able to sustain any further lateral loading, similar to the so-called short-pile failure mode described by Broms (1964a, 1964b). This assumption must always be checked. Once the limiting state of stress acting on the shaft has been determined, the calculated maximum bending moment and shear force in the shaft should be compared to the capabilities of the reinforced concrete section. If either of these calculated values exceeds the appropriate section properties, then failure will be governed by the strength of the shaft itself.

To determine the ultimate lateral loads acting on a short shaft, the distribution of the limiting reaction (force per unit length acting on the shaft) is required. This distribution may be evaluated as follows.

When a lateral load is applied at the rock surface, the rock mass immediately in front of the shaft will undergo zero vertical stress, while horizontal stress is applied by the leading face of the shaft. Ultimately, the horizontal stress may reach the uniaxial compressive strength of the rock mass and, with further increases in the lateral load, the horizontal stress may decrease as the rock mass softens during postpeak deformation. Large lateral deformations may be required for the rock mass at depth to exert a maximum reaction stress on the leading face of the shaft. Therefore, it is reasonable to assume that the reaction stress at the rock mass surface, in the limiting case of loading of the shaft, is zero or very nearly zero as a result of the postpeak softening. Along the sides of the shaft, some shearing resistance may be mobilized, and this is likely to be approximately the same as the maximum unit side resistance under axial compression, \( \tau_{\text{max}} \).

At greater depth, it is reasonable to assume that the stress in front of the shaft may increase from the initial in situ horizontal stress level, \( \sigma_{\text{h0}} \), to the
limit stress, $p_L$, reached during the expansion of a long cylindrical cavity, i.e., the plane strain condition will apply. Behind the shaft, the horizontal stresses will decrease, and after tensile rupture of the bond between the concrete and the rock mass, the horizontal stress will reduce to zero. At the sides of the shaft, some shearing resistance may also be mobilized. Therefore, at depth, the ultimate force per unit length resisting the lateral loading is likely to be about $B(p_L + \tau_{\max})$.

Closed-form solutions have been found for the limit stresses developed during the expansion of a long cylindrical cavity in an elastoplastic, cohesive-frictional, dilatant material (Carter et al. 1986). This limit stress ($p_L$) may be determined from the following parametric equation in the nondimensional quantity $\rho$ (Carter et al. 1986):

$$\frac{2G_r}{\sigma_{hi} + c_r \cot \varphi_r} = \left(\frac{N - 1}{N + 1}\right) \left(Tp^n - Z\rho\right) \quad \text{(21)}$$

with

$$\rho = \frac{p_L + c_r \cot \varphi_r}{\sigma_R + c_r \cot \varphi_r} \quad \text{(22)}$$

in which

$$T = 2\left(1 + \frac{x}{\alpha + \beta}\right) \quad \text{(23)}$$

$$Z = 2\left(\frac{x}{\alpha + \beta}\right) \quad \text{(24)}$$

$$\sigma_R = \left[\left(\frac{2N}{N + 1}\right)\left(\sigma_{hi} + c_r \cot \varphi_r\right)\right] - c_r \cot \varphi_r \quad \text{(25)}$$

$$\alpha = \frac{1}{L} \quad \text{(26)}$$

$$\beta = \frac{1}{N} \quad \text{(27)}$$

$$n = \frac{1 + \alpha}{1 - \beta} \quad \text{(28)}$$

$$L = \frac{1 + \sin \psi_r}{1 - \sin \psi_r} \quad \text{(29)}$$

$$N = \frac{1 + \sin \varphi_r}{1 - \sin \varphi_r} \quad \text{(30)}$$

$$\chi = \frac{(1 - \nu_r)(1 + LN) - \nu_r(L + N)}{LN} \quad \text{(31)}$$

and $G_r$ = elastic shear modulus; $\nu_r$ = Poisson ratio; $c_r$ = cohesion intercept; $\varphi_r$ = friction angle; and $\psi_r$ = dilation angle, all of the rock mass. The rock
FIG. 8. Limit Solution for Expansion of Cylindrical Cavity
mass is assumed to obey the Mohr-Coulomb failure criterion, and dilatancy accompanies yielding according to the following flow rule (Davis 1968): 

$$\frac{d\varepsilon_{3p}}{d\varepsilon_{1p}} = -L \quad \text{.................................................. (32)}$$

in which $d\varepsilon_{1p}$ and $d\varepsilon_{3p}$ = the major and minor principal plastic strain increments, respectively.

In most practical cases, the in situ horizontal stress, $\sigma_{hi}$, will be small compared to the cohesion, $c_r$, and, therefore, (21) can be simplified slightly by substitution of $\sigma_{hi} = 0$. For convenience, solutions for the limit pressures ($p_L$) have been plotted in Fig. 8 (Carter et al. 1986) for selected values of $\nu_r$, $\varphi_r$, and $\psi_r$. The central vertical axis on each plot indicates the ratio of the plastic radius at the limit condition ($R$) to the cavity radius ($a$). These charts may be used by entering with a value of $G_r(c_r \cot \varphi_r)$ and working clockwise around the figure, determining in turn values of $R/a_r$, then $p_L = (p_L + c_r \cot \varphi_r)/(\sigma_R + c_r \cot \varphi_r)$, and, thus, determining the limit pressure $p_L$.

One final problem remains, and that is determining the depth at which this limit stress is mobilized. In an earlier study by Randolph and Houslby (1984), it was suggested that, in a cohesive material, this depth would be about 3 shaft diameters ($3B$). In the absence of any other data, this suggestion will be adopted. Therefore, the proposed distribution of ultimate force per unit length resisting the shaft is as shown in Fig. 9.

The ultimate lateral force that may be applied for the conditions given is

$$H_{ult} = \left(\frac{p_L D}{6} + \bar{t}_{\text{max}} B\right)D \quad \text{for } D < 3B \quad \text{.......................... (33a)}$$

$$H_{ult} = \left(\frac{p_L}{2} + \bar{t}_{\text{max}}\right)3B^2 + \left(p_L + \bar{t}_{\text{max}}\right)(D - 3B)B$$

for $D > 3B \quad \text{.......................... (33b)}$
The maximum bending moment in the shaft is then calculated from $H_{ult}$ and the reaction distribution shown in Fig. 9. If the lateral loading consists of a horizontal force ($H$) and an applied moment ($M$), then, for purposes of calculating an appropriate bending moment distribution, these may be represented by an equivalent force of the same magnitude but applied at a height ($e = M/H$) above the rock mass surface.

The theoretical approach suggested previously [(33)] may be used to calculate the ultimate lateral load for a rock-socketed pier if suitable data are available for the rock mass strength and deformation parameters $c_r$, $\phi_r$, $\gamma_r$, $\tau_{max}$, $G_r$, and $v_r$. This method for predicting the ultimate capacity should be used with some caution. It has yet to be tested against a field test data base, and until this is done it must only be regarded as providing tentative estimates.

**LATERAL LOAD TESTS**

There is a dearth of data in the literature on the performance of rock-socketed shafts subjected to lateral loading. Indeed, the authors have been able to find only one case that was reported by K. O. Tucker and S. Askari (Southern California Edison Co., Los Angeles, Calif., unpublished internal report, 1986), and it involved the jacking together of two adjacent shafts, which once formed part of the foundations for an electrical transmission line structure. A schematic cross section, showing the two shafts and the loading arrangement, is given in Fig. 10. Note that each shaft has a different length and diameter and, in particular, that each has a different eccentricity of the loading point above the immediate rock surface. Furthermore, between the shafts, the rock surface slopes at approximately $25^\circ$ to the horizontal. This last feature will be ignored in the interpretation of the test data, and the immediate surface around each shaft will be treated as if it were
horizontal. Structural interaction between the shafts and the fact that shaft 14-U has an enlarged base also have been ignored. However, the eccentricity of loading of each shaft has been taken into account, as have the eccentricities of the heights above the rock surface of the displacement measuring points. In the latter regard, each shaft has been assumed to behave rigidly.
TABLE 1. Details of Lateral Load Test

<table>
<thead>
<tr>
<th>Shaft identification</th>
<th>D (m)</th>
<th>B (m)</th>
<th>e (m)</th>
<th>S' (MN/m)</th>
<th>S (MN/m)</th>
<th>G* (MN/m²)</th>
<th>E_r (MN/m²)</th>
<th>E_r/G* (B/2D)²</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-U</td>
<td>1.8</td>
<td>0.9</td>
<td>0.426</td>
<td>498</td>
<td>600</td>
<td>196</td>
<td>414</td>
<td>16</td>
</tr>
<tr>
<td>14-D</td>
<td>2.4</td>
<td>1.2</td>
<td>1.551</td>
<td>149</td>
<td>157</td>
<td>48</td>
<td>101</td>
<td>65</td>
</tr>
</tbody>
</table>

during lateral loading, and the rock mass was assumed to behave elastically, so that horizontal displacements at the groundline may be calculated from the measured horizontal displacements using the centers of rotation deduced according to (10). For the assumption of rigid shaft behavior to be valid, the condition given by (7b) must hold.

The measured lateral force-horizontal displacement relationship for each of these shafts is given in Figs. 11 and 12. Note the initial linearity of the load-deflection curves, which suggests the use of linear elastic theory, at least at working load levels. A straight line has been fitted to the initial portion of each curve, and the slope of the line $S'$ has been reported in Table 1 for each case. Also recorded in Table 1 are the slopes $S$ of the linear relations between the measured applied load and the deduced horizontal displacements at the groundline. Eq. (8) has been used to determine values of $G^*$ for each case from the slopes ($S = H/u$), and the known geometries and loading configurations. In each case, a value of $v_r = 0.25$ has been assumed for the rock mass, allowing the determination of $E_r$ from (2) and (3). Deduced values of the Young’s modulus for the rock mass immediately surrounding each shaft have also been listed in Table 1. It can be seen that the calculated modulus for shaft 14-U is approximately four times that for shaft 14-D. Values of the bending rigidity of the shafts have not been given in the source reference, but, assuming typical reinforced concrete details, $E_e$ is estimated to be about 50 GN/m² ($7 \times 10^6$ psi). The relative stiffness, $(E_e/G^*)(B/2D)^2$, has been calculated for each shaft, assuming $E_e = 50$ GN/m², and is also given in Table 1. In both cases, the relative stiffness is less than 100, indicating that for each shaft the assumption of rigid behavior may be slightly inaccurate [see (7b)]. However, values of $(E_e/G^*)(B/2D)^2 = 16$ and 65 still indicate very stiff shafts.

CONCLUSIONS

A method of analysis has been described that allows the prediction of the behavior at working load levels of a shaft socketed in rock, when the shaft is subjected to prescribed lateral forces and moments acting at the groundline. A simple model for the mechanical behavior has been suggested, and this model has allowed closed-form predictions of the response of a shaft.

The application of the simple model to the interpretation of a field loading test reported in the literature also has been described. However, there are very few field data from lateral load tests on drilled shafts in rock. Further research in this area is required to understand better this mode of behavior.

A method of analysis for estimating the lateral load capacity of a rock-socketed shaft also has been tentatively proposed. Further field testing also is required to assess the reliability of this prediction method.
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APPENDIX. REFERENCES


